The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Simply put, if f is continuous on the interval [a,b] we can find the sum of the shaded regions by evaluating the indefinite integral at b and a and subtracting the values. This give us the "net change"in the dependent (y-values) of the integral.

Demo **Demo 2** **Note**

The Fundamental Theorem of Calculus

$$\int_a^b |f(x)|dx$$

Rather than the net change, the definite integral of the absolute value of f(x) would represent the sum of an only increasing function. This would mean that the value would be a total change rather than a net change. This is often used to find total distance traveled given velocity.

Demo

Demo 2

Note

Practice

$$\int_{-1}^{3} (-x^2 + 1)dx$$

1 2 3 4 5 6 7

Practice

$$\int_0^5 \frac{x-2}{3}\,dx$$

1 2 3 4 5 6 7

Practice

$$\int_{2}^{4} \sqrt{m}(m^2 - 3)dm$$

1 2 3 4 5 6 7

Practice

$$\int_{-3}^{7} |z^2 - 3z - 10| \, dz$$

1 2 3 4 5 6 7

Practice

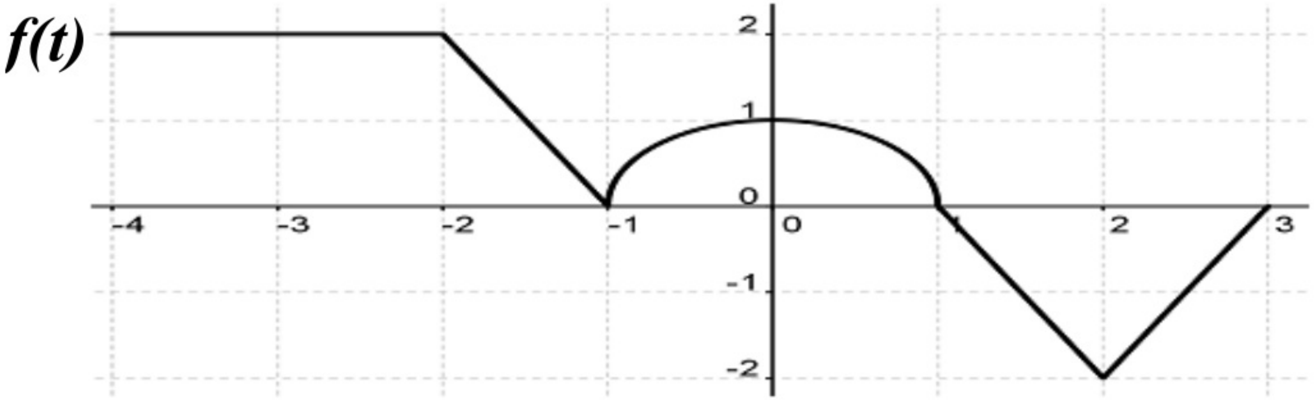
The function v(t) = 2cost - sint models a particles velocity as it travels along a line from time t = 0 to t = 2p. What is the particles distance from its starting position at time t = p? What is the total distance traveled by the particle in that time?

Practice

Given that $\int_{-6}^{3} f(x)\,dx = 10$ **and F(3) = 2, find F(-6)**

1 2 3 4 5 6 7

The graph of *f(t)* is below. Let $g(x) = \int_{-4}^{x} f(t)\,dt$



a) Find g(-2), what does this mean in terms of f(t)?
b) On what interval is g increasing?
c) Does g have any relative extrema? If so, what is the coordinate?

①②③④⑤⑥⑦

Average Value of a Function on an Interval

$$\frac{1}{b-a}\int_a^b f(x)dx$$

We can use this to calculate the average rate when we start with a rate function. How do we find the average rate when we start with the integral of a rate function?

P1 2

Average Value of a Function on an Interval

Find the average value of $f(x) = x^{1/2} + 2$ on [0,4]

P1 2

Average Value of a Function on an Interval

The rate, p(t), at which a company's profit increased per month, t, can be modeled by $p(t) = \frac{1}{2}t^3 - \sqrt[3]{t} + 3$ where p(t) is in hundreds of dollars. What was the average rate of increase for the first two months? After how many months was the rate equal to the average?

P1 2

What happens when the upper limit of integration is a variable?

The integral is a function of x

$$F(x) = \int_a^x f(t)\,dt$$

The integrand is a funtion of t

When this occurs we can still apply the first fundamental theorem of calculus. The only difference is we will be evaluating *F* with a constant and a variable so our solution will be a function rather than a value.

1

What happens when the upper limit of integration is a variable?

Find *F* as a function of x and evaluate it at

x $= 0, \frac{\pi}{2}, \frac{2\pi}{3}$

$$F(x) = \int_0^x \sec x \tan x \, dx$$

1

The Second Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,\mathrm{dt} = f(x)$$

1

2

If f is continuous on an interval, the derivative of the definite integral of f with respect to an upper limit variable is equivalent to f in terms of the variable of differentiation. What's the derivative if the upper and lower limits are constants?

The Second Fundamental Theorem of Calculus

Find F'(x) in each case below:

a) $F\left(x\right) = \int_5^x \sqrt{s}\, ds$

b) $F(x) = \int_0^x \tan t \, dt$

1

2

The Second Fundamental Theorem of Calculus

Find F' for the following:

a) $F(x) = \int_4^{x^3} t^4 \, dt$

b) $F(x) = \int_0^{\cot x} \sin\theta \, d\theta$

c) $F(x) = \int_{2x-1}^{x^3} (2m + 1) \, dm$

d) $F(x) = \int_x^{4x^2} \sqrt{t} \, dt$

1

2